On the measurement of $\chi_2(^3P_2)$ quarkonium state in the processes $e^+ + e^- \to \bar{p} + p$ and $\bar{p} + p \to e^+ + e^-$

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The intermediate quarkonium state $\chi_2(^3P_2)$ in electron-positron annihilation to proton and antiproton as well as in antiproton-proton annihilation to electron and positron can produce backward-forward asymmetry, when populated through two photon exchange. We use the dispersion relation method, which permits to express the asymmetry in terms of partial widths of quarkonium decay. The asymmetry dependence on the center of mass energy in the range near the resonance is presented. The comparison with a similar effect in these reactions with the neutral Z-boson in the intermediate state is given. We show that these effects are $\leq 10^{-3}$. The main source of asymmetry is of pure QED origin ($\sim 10^{-2}$) which arises from the interference between initial and final state real photon emission.

I. INTRODUCTION

In a recent paper [1] it was proposed that a large C-odd effect could arise in the electron-positron annihilation into a hadron pair through two photon exchange which couple to resonances such as $\Phi = \eta, \eta_C, \chi_0, \chi_2, \rho, a_1$. In one loop level, taking into account the two-photon intermediate state (Figs. 1(a) and 1(b)), there is the possibility to access the $^1S_0, ^3P_0, ^3P_2$ quarkonium intermediate states, which are bound states of charm quark and antiquark. It was shown in Refs. [1, 2] that the contribution to the asymmetry arising from the intermediate states $^1S_0, ^3P_0$ is proportional to the ratio of the electron mass to the proton mass and it is negligible. The intermediate state with quantum numbers $^3P_2(3556) = \chi_2$ can be produced by two virtual photons, which arise from the annihilation subprocess of electron and positron. In Ref. [1] this contribution to asymmetry was calculated and it was indicated that it is responsible for a large asymmetry, which can reach up to 40%. In this case, two photon exchange would be experimentally observable in present and planned experiments.

In principle two photon exchange is suppressed by the factor $\alpha = 1/137$, the electromagnetic fine structure constant. Therefore, specific mechanisms should be advocated, which can compensate such suppression.
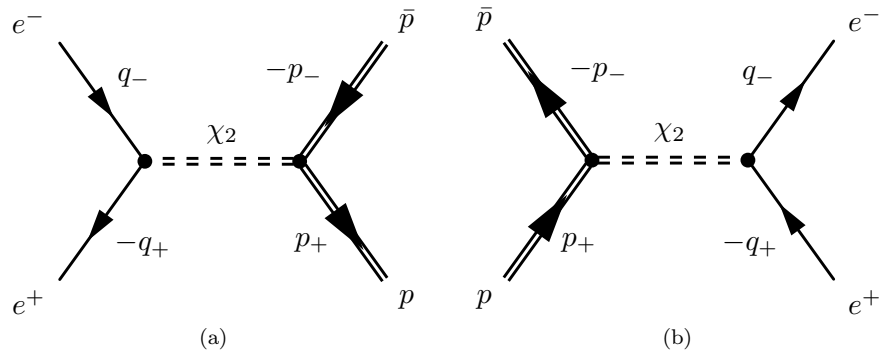

FIG. 1: Feynman diagrams of processes $e^+ + e^- \to \bar{p} + p$ and $\bar{p} + p \to e^+ + e^-$ with quarkonium intermediate states.

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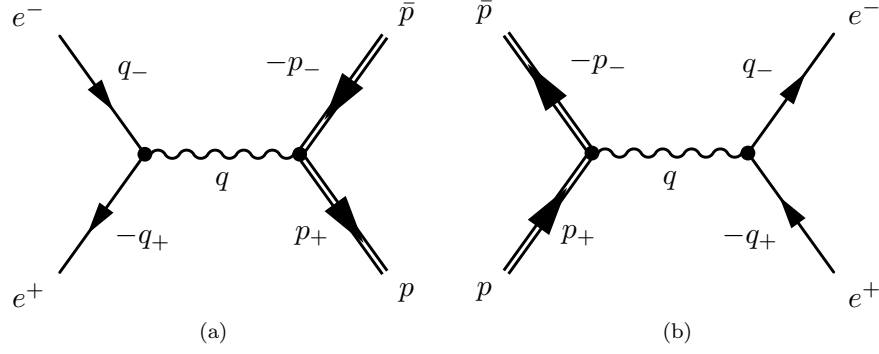[§]Electronic address: etomasi@cea.fr

FIG. 2: Feynman diagrams of processes $e^+ + e^- \to \bar{p} + p$ and $\bar{p} + p \to e^+ + e^-$ in Born approximation.

In this note we recalculate the excitation of the χ_{c2} resonance, through two photon exchange, and show that the asymmetry in this case does not exceed level of 10^{-3}. We compare this value with the asymmetry generated by Z-boson exchange, which we find also of the order of 10^{-3}, and by the asymmetry of pure QED nature, due to the interference of photon emission in initial and final state [3], which turns out to be the most important source of asymmetry ($\sim 10^{-2}$).

II. BORN APPROXIMATION

At the facilities VEPP (Novosibirsk), BEPC (Beijing), and PANDA/FAIR (Darmstadt), one can measure the process of creation of proton and anti-proton in electron-positron annihilation:

$$e^+(q_+) + e^-(q_-) \to \bar{p}(p_-) + p(p_+) \tag{1}$$

and the process of annihilation of proton and antiproton into electron and positron

$$\bar{p}(p_-) + p(p_+) \to e^+(q_+) + e^-(q_-) \tag{2}$$

in the energy range of the mass of the quarkonia bound states of charmed quark and anti-quark. In Born approximation the amplitude of these processes has the form (see Figs. 2(a) and 2(b)):

$$\mathcal{M}_\gamma = \frac{4\pi\alpha}{s} \left[\bar{v}(q_+)\gamma^\mu u(q_-) \right] \left[\bar{u}(p_+)\Gamma_\mu(q)v(p_-) \right], \tag{3}$$

where $q = p_+ + p_- = q_+ + q_-$, $s = q^2$ and electromagnetic vertex of the proton $\Gamma_\mu(q)$ is parametrized in terms of two form factors:

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{1}{4M_p} \left(\slashed{q}\gamma_\mu - \gamma_\mu\slashed{q} \right) F_2(q^2), \tag{4}$$

where F_1 and F_2 are the Dirac and Pauli form factors of the proton in the time-like region of momentum transfer $(q^2 > 4M_p^2)$ which related with the Sachs electric and magnetic proton form factors as

$$G_E = F_1 + \frac{s}{4M_p^2}F_2, \qquad G_M = F_1 + F_2. \tag{5}$$

The square modulus of the matrix element (3) has the form

$$\sum_{spins} |\mathcal{M}_\gamma|^2 = 4(4\pi\alpha)^2 \left[|G_M|^2(1 + c^2) + (1 - \beta^2)|G_E|^2(1 - c^2) \right], \qquad \beta = \sqrt{1 - \frac{4M_p^2}{s}}, \tag{6}$$

where β is the proton velocity and $c \equiv \cos\theta$, θ is the angle between vectors $\mathbf{q}_-$ and $\mathbf{p}_+$ in the center of mass system of the initial particles. The cross sections of processes (1) and (2) in Born approximation thus has the form:

$$\frac{d\sigma^{p\bar{p} \to e^+e^-}}{dc} = \frac{1}{8s\beta}\frac{1}{16\pi}\sum_{spins}|\mathcal{M}_\gamma|^2, \qquad \frac{d\sigma^{e^+e^- \to p\bar{p}}}{dc} = \frac{1}{8s}\frac{\beta}{16\pi}\sum_{spins}|\mathcal{M}_\gamma|^2. \tag{7}$$
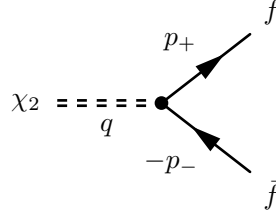
FIG. 3: Feynman diagrams of the conversion of χ_2 tensor particle into fermion–antifermion pair.

The interference of the Born amplitude $\mathcal{M}_\gamma$ from (3) with a single photon in the intermediate state with the amplitude with quarkonium in the intermediate state (see Figs. 1(a) and 1(b)) originates backward-forward asymmetry

$$\mathcal{A} = \frac{d\sigma(c) - d\sigma(-c)}{d\sigma(c) + d\sigma(-c)}, \tag{8}$$

where $d\sigma(c) = d\sigma/dc$ is the angular dependent cross section. Let us estimate this asymmetry.

III.   ASYMMETRY FROM χ_2 : APPROXIMATION OF PARTIAL WIDTHS

The amplitude of the processes (1) and (2) with the χ_{c2} intermediate state corresponds to the diagrams on Figs. 1(a) and 1(b) and can be written as:

$$\mathcal{M}_{\chi_2} = \frac{\mathcal{M}_{\chi_{c2} \to e^+ e^-}\, \mathcal{M}_{\chi_{c2} \to p\bar{p}}}{s - M_{\chi_2}^2 + i M_{\chi_2}\Gamma_{\chi_2}}, \tag{9}$$

where we used the "partial width" approximation which is acceptable within the χ_{c2}–resonance intermediate state vicinity. The matrix elements $\mathcal{M}_{\chi_{c2} \to e^+ e^-}$ and $\mathcal{M}_{\chi_{c2} \to p\bar{p}}$ describe the conversion of χ_2 tensor particle into the electron–positron and proton–antiproton pairs correspondingly. This type of matrix elements can be parametrized in the following way [4] (see Fig. 3):

$$\mathcal{M}_{\chi_2 \to f\bar{f}} = g_f \left[\bar{v}(p_+)\gamma^\mu u(p_-)\right](p_+ - p_-)^\nu \chi_{\mu\nu}, \tag{10}$$

where g_f is the corresponding coupling constant and $\chi_{\mu\nu}$ is the polarization tensor of $J = 2$ state, which has the following properties:

$$\begin{aligned}
\chi_{\mu\nu} = \chi_{\nu\mu}, \qquad \chi_{\mu\nu}g^{\mu\nu} = 0, \qquad \chi_{\mu\nu}q^\mu = 0, \\
\chi_{\mu\nu}\chi_{\lambda\sigma} = \frac{1}{2}\left(\Pi_{\mu\lambda}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\lambda}\right) - \frac{1}{3}\Pi_{\mu\nu}\Pi_{\lambda\sigma}, \\
\Pi_{\alpha\beta} = -g_{\alpha\beta} + \frac{q_\alpha q_\beta}{q^2}.
\end{aligned} \tag{11}$$

The decay width which corresponds to the amplitude $\mathcal{M}_{\chi_2 \to f\bar{f}}$ has the form:

$$\Gamma_{\chi_2 \to f\bar{f}} = \frac{g_f^2\, M_{\chi_2}^3}{120\pi}\beta_f^3(5 - 2\beta_f^2), \qquad \beta_f = \sqrt{1 - \frac{4m_f^2}{M_{\chi_2}^2}}, \tag{12}$$

where β_f is the velocity of final fermion f. In particular

$$\Gamma_{\chi_2 \to p\bar{p}} = \frac{g_p^2\, M_{\chi_2}^3}{120\pi}\beta^3(5 - 2\beta^2), \qquad \Gamma_{\chi_2 \to e\bar{e}} = \frac{g_e^2\, M_{\chi_2}^3}{40\pi}. \tag{13}$$

Keeping in mind possible complexity of the product $g_e g_p = |g_e g_p|\, e^{i\phi}$, we can write the interference of Born amplitude $\mathcal{M}_\gamma$ (see (3)) with the amplitude $\mathcal{M}_{\chi_2}$ from (9) which gives the following result:

$$|\mathcal{M}|_{\text{int}}^2 = 2\sum_{\text{spins}} \text{Re}\left[\mathcal{M}_\gamma^* \mathcal{M}_{\chi_2}\right] = \frac{32\pi\alpha M_{\chi_2}^3}{\Gamma_\chi}|g_e g_p|\frac{y\cos\phi + \sin\phi}{y^2 + 1}\beta c(1 - \beta^2 c^2), \tag{14}$$

where y is the convenient variable which vanishes at the resonance region:

$$y = \frac{s - M_{\chi2}^2}{M_{\chi2}\Gamma_{\chi2}}. \tag{15}$$

Thus we can estimate the asymmetry $\mathcal{A}_\chi$ from (8) as

$$\mathcal{A}_\chi = \mathcal{A}_0 \frac{y\cos\phi + \sin\phi}{y^2 + 1} \frac{\beta c(1 - \beta^2 c^2)}{2 - \beta^2(1 - c^2)}, \qquad \mathcal{A}_0 = \frac{|g_e g_p| M_{\chi2}^3}{\pi\alpha\Gamma_{\chi2}}, \tag{16}$$

where we used point-like proton approximation ($F_1 = 1$, $F_2 = 0$).

Unfortunately the partial decay width of the decay of χ_2 state into the electron–positron pair as well as the phase ϕ is not known. Below we will calculate these phases in approximation of two photon and two gluon intermediate state mechanisms, for the conversion of χ_2 into electron-positron pair and into proton-anti-proton pair.

IV.   TWO PHOTON AND TWO GLUON INTERMEDIATE STATE APPROACH

Our approach for evaluating the conversion amplitudes consists in the calculation of their s-channel discontinuities and in the restoration of the whole amplitude by means of dispersion relations. In this way we can express the results in terms of the experimentally measurable partial width of conversion of χ_2 tensor meson state to two photons and to hadrons.

We use the partial width approximation for the total amplitude $\mathcal{M}_\chi$ again (9) and calculate the imaginary part as:

$$\Delta |\mathcal{M}|_{\text{int}}^2 = 2 \sum_{\text{spins}} \text{Re} \left[\mathcal{M}_\gamma^* \Delta\mathcal{M}_{\chi2}\right]. \tag{17}$$

The amplitudes $\mathcal{M}_{\chi c2 \to e^+ e^-}$ and $\mathcal{M}_{\chi c2 \to p\bar{p}}$ are estimated in the approximation of two photon and two gluon intermediate state mechanisms:

$$\mathcal{M}_{\chi c2 \to e^+ e^-} = 4(4\pi\alpha)^2 g_\gamma \chi_{\alpha\beta} T_{\alpha\beta\lambda}$$
$$\mathcal{M}_{\chi c2 \to p\bar{p}} = 4(4\pi\alpha_s) g_g c_{col} \chi_{\mu\nu} R_{\mu\nu\lambda} \tag{18}$$

where the dimensional constants g_γ, g_g describe the conversion of $^3P_2$ state to two photons and two gluons. In Eq. (18) we include the color factor $I c_{col} = 3(1/2)\delta^{ab}t^a t^b = 2I$, where I is the unit matrix in the color space, which describes the interaction of two gluons with each of the quark of the proton. Moreover we define:

$$T_{\alpha\beta\lambda} = \frac{1}{4} Sp\hat{q}_- \gamma_\lambda \hat{q}_+ O_{\alpha\beta}^e,$$

$$O_{\alpha\beta}^e = \int d\Phi_2 \frac{1}{2q_- k_1} K_{\alpha\beta\rho\sigma} \gamma_\rho(\hat{q}_- - \hat{k}_1)\gamma_\sigma;$$

$$R_{\mu\nu\lambda} = \frac{1}{4} Sp(\hat{p}_- - M_p)\Gamma_\lambda(\hat{p}_- - M_p)O_{\mu\nu}^p,$$

$$R_{\mu\nu}^p = \int d\Phi_2 \frac{1}{2p_+ k_1} K_{\mu\nu\rho\sigma} \gamma_\rho(\hat{p}_+ - \hat{k}_1 + M_p)\gamma_\sigma, \tag{19}$$

with

$$K_{\alpha\beta\rho\sigma} = (k_1 k_2) g_{\alpha\sigma} g_{\beta\rho} + k_{1\alpha} k_{2\beta} g_{\rho\sigma} - k_{1\alpha} k_{2\sigma} g_{\rho\beta} - k_{1\rho} k_{2\beta} g_{\alpha\sigma}. \tag{20}$$

The corresponding matrix elements and the partial widths are

$$M_{2\gamma} = g_\gamma K_{\mu\nu\alpha\beta} \chi_{\alpha\beta} e_1^\mu(k_1) e_1^\nu(k_2),$$

$$\Gamma_{\chi2 \to \gamma\gamma} = \frac{g_\gamma^2 M_\chi^3}{320\pi};$$

$$M_{gg} = g_g \frac{1}{2}\delta^{ab} K_{\mu\nu\alpha\beta} \chi_{\alpha\beta} e_1^{a\mu}(k_1) e_1^{b\nu(k_2)},$$

$$\Gamma_{\chi2 \to gg} = \frac{g_g^2 M_\chi^3}{160\pi}. \tag{21}$$

Keeping in mind the duality principle, we assume that the $\chi_2 \to hadrons$ decay goes through two gluons at the leading order of α_s. Thus we can identify $\Gamma_{\chi_2 \to 2gluons}$ with the hadronic width of $^3P_2$ state

$$\Gamma_{\chi_2 \to gg} = 0.8\Gamma_{total} = 1.6 \text{ MeV}, \Gamma_{\chi_2 \to \gamma\gamma} = 0.2\Gamma_{total} = 0.4 \text{ MeV}, \tag{22}$$

where the factor $0.8(0.2)$ takes into account the proportion of the hadronic(photonic) decay channels of χ_2 [5]. This gives

$$g_\gamma = 3.3857 \cdot 10^{-4} \text{ GeV}^{-1}, \qquad g_g = 0.13 \text{ GeV}^{-1}. \tag{23}$$

The phase factor $e^{i\phi}$ is defined as $e^{i\phi} = e^{i(\phi_e + \phi_p)}$ where $e^{i\phi_e}$ and $e^{i\phi_p}$ arise respectively from the amplitudes $\mathcal{M}_{\chi c2} \to e^+ e^-$ and $\mathcal{M}_{\chi c2} \to p\bar{p}$.

A. Two photon intermediate state to the vertex $\chi_2 \to e\bar{e}$

For discontinuity of amplitude of conversion of the tensor state to the lepton pair through two-photon intermediate state T we obtain

$$\Delta_s T_{\alpha\beta\lambda} = 4\pi\alpha g_\gamma \int d\Phi_2 \frac{1}{2(k_1 q_-)} K_{\alpha\beta\sigma\rho} \frac{1}{4} \text{Sp} \left[\not{q}_- \gamma_\lambda \not{q}_+ \gamma_\rho \left(\not{q}_- - \not{k}_1 \right) \gamma_\sigma \right]. \tag{24}$$

Performing the integration on two photon volume

$$d\Phi_2 = \int \frac{d^3 k_1 d^3 k_2}{2\omega_1 2\omega_2 (2\pi)^2} \delta^4(q - k_1 - k_2), \tag{25}$$

we obtain (see details in Appendix A)

$$T_{\alpha\beta\lambda} = \frac{5\alpha}{72\pi} g_\gamma \left(\ln \frac{M_{\chi_2}^2}{m_e^2} - i\pi \right) \frac{1}{4} \text{Sp} \left[\not{q}_- \gamma_\lambda \not{q}_+ \gamma_\alpha \right] (q_+ - q_-)_\beta. \tag{26}$$

We can conclude that the phase associated with conversion to two photons ϕ_e is small

$$tg\phi_e = \frac{\pi}{\ln \frac{M_{\chi_2}^2}{m_e^2}} = 0.17. \tag{27}$$

So we assume further the conversion amplitude of $^3P_2$ state to the lepton pair to be real (i.e., $\phi_e \approx 0$).

B. Total discontinuity, real part and asymmetry

After conversion of tensor structures, omitting the phase associated with lepton part of the amplitude and performing the angular integration (see Appendix B) we obtain for the total discontinuity (we are interested only in the odd part of the contribution)

$$|\mathcal{M}|_{int}^2 = \frac{10\pi}{27} \alpha^2 \alpha_s (M_{\chi_2}^2 g_\gamma g_g) \ln \frac{M_{\chi_2}^2}{m_e^2} \frac{s}{(s - M_{\chi_2}^2)^2 + M_{\chi_2}^2 \Gamma^2} \left[(s - M_{\chi_2}^2) H_{odd} + M_{\chi_2} \Gamma G_{odd} \right],$$

$$G_{odd} = \frac{c}{\beta^3} \left[-34\beta^4 - 6\beta^3 + 54\beta^2 - 18 + 9\frac{(1 - \beta^2)^3}{\beta} L \right] +$$

$$+ c^2 \left[54\beta^4 + 10\beta^3 - 6\beta^2 + 30 + \frac{3}{\beta} \left(4\beta^5 - 6\beta^3 + 7\beta - 5 \right) L \right], \qquad L = \ln \frac{1 + \beta}{1 - \beta}, \tag{28}$$

where we used the approximation of point-like proton (i.e. $F_1 = 1$ and $F_2 = 0$) which is realistic near the threshold. The real part, corresponding to G_{odd}, can be found using the dispersion relation (see also comments in Appendix B):

$$H_{odd}(\beta, c) = \frac{1}{2\pi} \left\{ G_{odd}(\beta, c) \ln \frac{1 - \beta^2}{\beta^2} + \int_0^1 \frac{2x dx}{x^2 - \beta^2} \left(-G_{odd}(\beta, c) + G_{odd}(x, c) \right) \right\}. \tag{29}$$
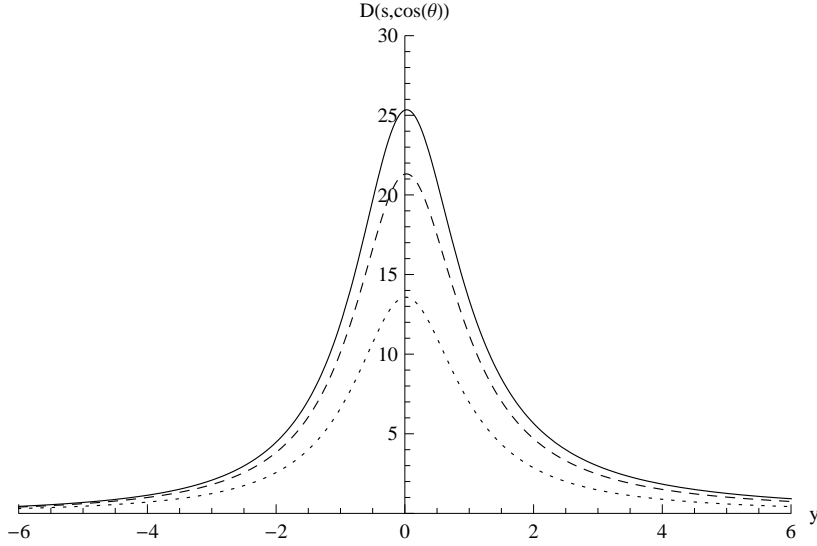
FIG. 4: The numerical estimation of the quantity $D(y, \cos(\theta))$ (see (30)) as a function of y within the region corresponding to $M_\chi - 3\Gamma_\chi \leq \sqrt{s} \leq M_\chi + 3\Gamma_\chi$. The solid line corresponds to $\theta = 10^o$, dashed line — $\theta = 30^o$, dotted line — $\theta = 50^o$.

Setting $F_1 = 1$, $F_2 = 0$, we obtain for the asymmetry

$$\mathcal{A}_\chi = \mathcal{A}_0 D, \qquad D = \frac{yH_{odd} + G_{odd}}{y^2 + 1} \frac{1}{2 - \beta^2(1 - c^2)}, \tag{30}$$

$$\mathcal{A}_0 = \frac{5\alpha_s}{864\pi}(M^2 g_\gamma g_g) \ln \frac{M_{\chi_2}^2}{m_e^2} = 1.86 \cdot 10^{-5}.$$

For the global phase we have

$$\phi(\beta, c) \approx \phi_p = \arctan \frac{G_{odd}(\beta, c)}{H_{odd}(\beta, c)}. \tag{31}$$

For the value $\beta = 0.85$ which corresponds to the $\sqrt{s} = M_\chi$ we find $\mathcal{A}_\chi \approx 5 \cdot 10^{-4}$. The quantity D as function of y, as defined in Eq. 15, is shown in Fig. 4 for different values of c, $c = \cos(\mathbf{q}_- \mathbf{p}_+)$. This quantity reaches its maximum value at the top of the resonance, where $y = 0$. The angular dependence of the quantity D is shown in Fig. 5 for different values of the total energy. One can see that D is largest at backward and forward angle, and rapidly falls when deviating from the top of the resonance.

The total phase ϕ, Eq. (31) is of the order of $-90°$ and shows a very weak dependence on the energy, see Fig. 6. The angular dependence of the total phase ϕ, Eq. (31), is illustrated in Fig. 7.

V. Z-BOSON AND QED CONTRIBUTIONS TO THE ASYMMETRY

For comparison we calculate the contribution to the asymmetry arising from the interference of two Born amplitudes, with photon and Z boson intermediate states (we set here $F_1 = 1, F_2 = 0$):

$$\mathcal{A}_Z = \mathcal{A}_0^Z \Psi_Z(c), \tag{32}$$

where

$$\mathcal{A}_0^Z = \frac{s}{M_Z^2 \sin^2(2\theta_W)} \approx 2.1 \cdot 10^{-3}, \qquad \text{at } \sqrt{s} = M_\chi, \tag{33}$$

$$\Psi_Z(c) = \frac{\beta c}{2 - \beta^2(1 - c^2)}, \tag{34}$$

with θ_W is the Weinberg angle, $\sin^2 \theta_W \approx 0.231$. The odd function $\Psi_Z(c)$ is shown in Fig. 8. One can see that this contribution, $\mathcal{A}_Z$ never exceeds 10^{-3}.
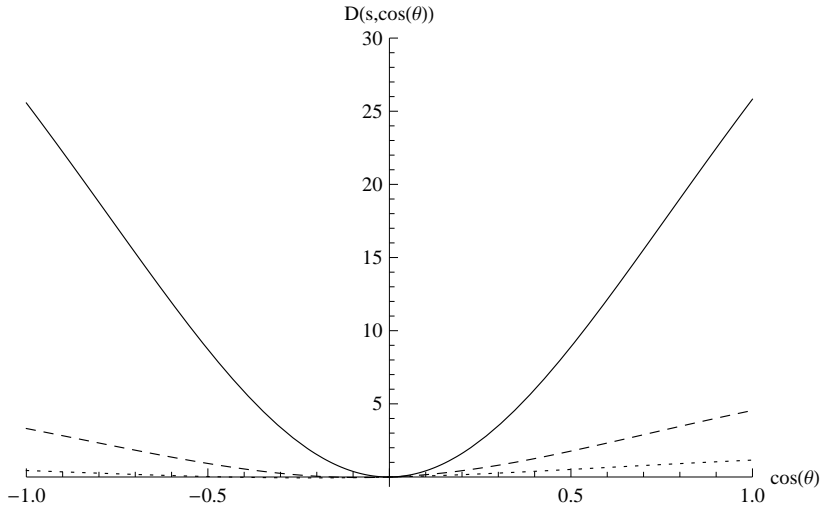
FIG. 5: The numerical estimation of the quantity $D(y, \cos(\theta))$ (see (30)) as a function of $c = \cos\theta$ for point-like proton (i.e. $F_1 = 1$, $F_2 = 0$). The solid line corresponds to $\sqrt{s} = M_\chi$, dashed line — $\sqrt{s} = M_\chi - \Gamma_\chi$, dotted line — $\sqrt{s} = M_\chi - 2\Gamma_\chi$.
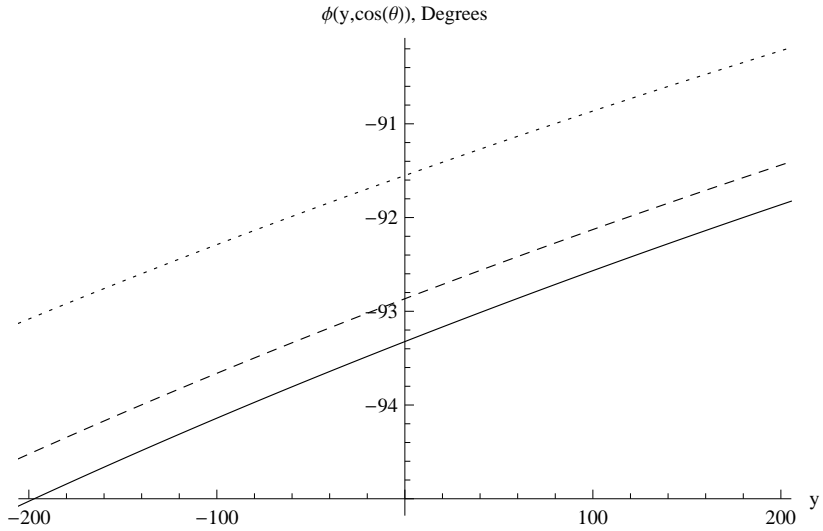

FIG. 6: The numerical estimation of the quantity $\phi(s)$ (in degrees) (see (31)) as a function of y within the region corresponding to $M_\chi - 100\Gamma_\chi \le \sqrt{s} \le M_\chi + 100\Gamma_\chi$. The solid line corresponds to $\theta = 10^o$, dashed line — $\theta = 30^o$, dotted line — $\theta = 50^o$.

Besides the two considered sources of the asymmetry, there is also a pure QED source, $\mathcal{A}_{QED}$. The total contribution to the odd part of the cross section was previously considered in Refs. [3, 6]. The angular dependence of the asymmetry corresponding to this calculation, is plotted for $\sqrt{s} = M_\chi$ in Fig. 9. The dashed curve corresponds to the interference of the Born amplitude with the box ones, and it is very small. The dominant contribution arises from the soft and hard real photon emission from initial and final states (solid line).

VI. CONCLUSION

We have calculated the charge asymmetry $\mathcal{A}$ for the reactions $p\bar{p} \leftrightarrow e^+e^-$, in the region of the χ_2 resonance, due to different mechanisms: two photon exchange $\mathcal{A}_\chi$, Z-boson exchange, $\mathcal{A}_Z$, and the pure QED mechanism, $\mathcal{A}_{QED}$.

Our conclusion is that

$$10^{-2} \sim |\mathcal{A}_{QED}| > |\mathcal{A}_Z| \ge |\mathcal{A}_\chi| \sim 10^{-4}. \tag{35}$$

We have focussed to the energy region concerned by the formation of the χ_2 resonance, because in a recent paper,
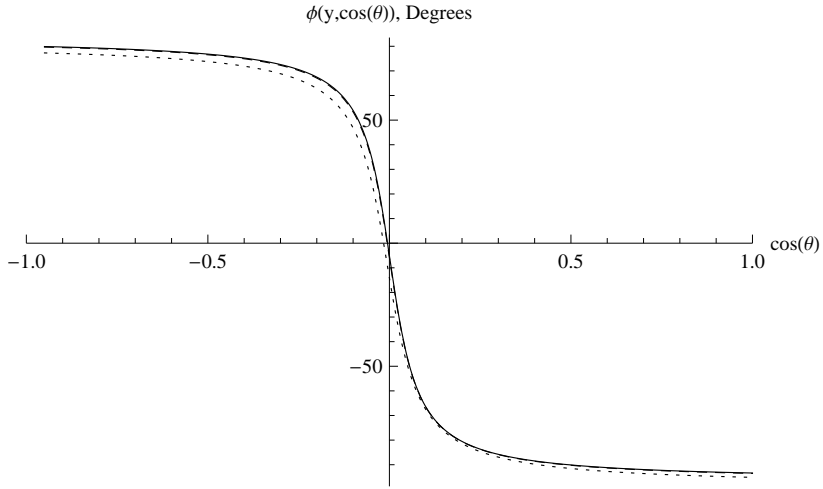
ϕ(y,cos(θ)), Degrees

FIG. 7: The numerical estimation of the quantity $\Psi(c)$ (see (31)) as a function of $c = \cos\theta$ for $\sqrt{s} = M_\chi$ and $F_1 = 1$, $F_2 = 0$. The solid line corresponds to $\sqrt{s} = M_\chi$, dashed line — $\sqrt{s} = M_\chi - 10\Gamma_\chi$, dotted line — $\sqrt{s} = M_\chi - 100\Gamma_\chi$.
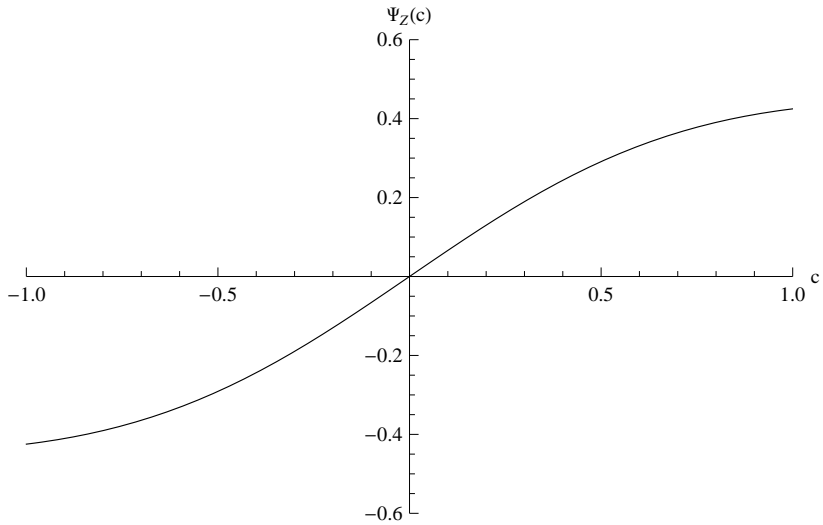

Ψ_Z(c)

FIG. 8: The numerical estimation of the quantity $\Psi_Z(c)$ (see (34)) as a function of $c = \cos\theta$ for $\sqrt{s} = M_\chi$.

Ref. [1], it was suggested that two photon exchange could make such asymmetry as large as 40%, depending strongly on the relative phase between electric and magnetic form factors. Such phase was not fixed by the calculation and the results were given for any possible value of the phase.

Our results do not confirm such finding. We estimated this phase within a model which describes the $\chi_2 \to p\bar{p}$ vertex with two gluon intermediate state which is assumed to saturate it. We show, indeed, that this asymmetry is maximum at the resonance, and at forward/backward angles, but its value keeps small ($\sim 10^{-4}$) due to the smallness of the constants. The coupling constants are expected to be small because such resonance is very narrow.

The possible reason of the discrepancy with Ref. [1] can be related to the different description of the vertex $\chi_2 \to p\bar{p}$. In our case two gluons insure the strong interaction nature of this vertex, which is proportional to α_s.

Let us note that the asymmetry measured in the process $p\bar{p} \to e^+e^-$, as accessible at the PANDA (Darmstadt) is expected to be the same as in $e^+e^- \to p\bar{p}$, which can be measuured at BES (Beijing), as well as at VEPP (Novosibirsk) and DAFNE (Frascati).
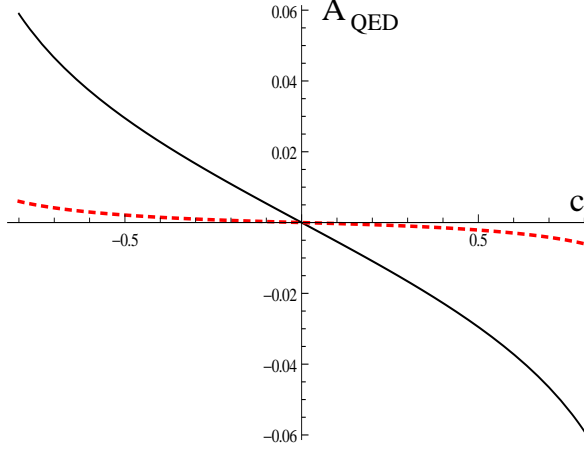
FIG. 9: Angular dependence of QED asymmetry in the χ_2 resonance region: interference Born-box amplitudes (dashed line), soft and hard real photon emission (solid line).

VII. ACKNOWLEDGEMENTS

Appendix A: The integrals for the amplitude of conversion of the $\chi_2(^3P_2)$ state into a lepton pair

Let us derive the relevant integrals the amplitude of conversion of the $\chi_2(^3P_2)$ state into a lepton pair.

The phase volume of the 2γ intermediate state can be written as:

$$\int \frac{d^4k_1}{(2\pi)^2}\delta(k_1^2)\delta(k_2^2) = \int \frac{d^3k_1 d^3k_2}{4\omega_1\omega_2(2\pi)^2}\delta^4(k_1+k_2-q) =$$
$$= \int \frac{d\omega d\Omega}{16\pi^2}\delta(\sqrt{s}-2\omega) = \frac{1}{16\pi}\int \frac{d\Omega_1}{2\pi}. \tag{A1}$$

Keeping in mind that $2(q_-k) = (s/2)(1-\beta c)$ and that $\int \frac{d\Omega}{2\pi} = \int\limits_{-1}^{1} dc$, we define:

$$\int \frac{d\Omega}{2\pi}\frac{1}{2q_-k}\{1; k_\mu; k_\mu k_\nu; k_\mu k_\nu k_\lambda\} = \{I; \alpha q_{-\mu} + \beta q\mu\,;$$
$$a_g g_{\mu\nu} + a_{--}q_{-\mu}q_{-\nu} + a_{qq}q_\mu q_\nu + a_{q-}(q_\mu q_{-\nu} + q_{-\mu}q_\nu);$$
$$a_{g-}(g_{\mu\nu}q_{-\lambda} + g_{\mu\lambda}q_\nu + g_{\nu\lambda}q_{-\mu}) + a_{gq}(g_{\mu\nu}q_\lambda + g_{\mu\lambda}q_\nu + g_{\nu\nu}q_\mu)$$
$$+ a_{---}q_{-\mu}q_{-\nu}q_{-\lambda} + a_{qqq}q_\mu q_\nu q_\lambda + a_{--q}(q_{-\mu}q_{-\nu}q_\lambda + q_{-\mu}q_\nu q_{-\lambda} + q_\mu q_{-\nu}q_{-\lambda})+$$
$$a_{-qq}(q_{-\mu}q_\nu q_\lambda + q_\mu q_{-\nu}q_\lambda + q_\mu q_\nu q_{-\lambda})\}. \tag{A2}$$

The calculation leads to:

$$I = \frac{2}{s}\ln\frac{s}{m_e^2}; \quad \alpha = I - \frac{4}{s}; \quad \beta = \frac{2}{s};$$
$$a_g = -\frac{1}{2}; \quad a_{--} = I - \frac{6}{s}; \quad a_{qq} = a_{q-} = \frac{1}{s};$$
$$a_{g-} = \frac{2}{g}; \quad a_{gq} = -\frac{1}{3}; \quad a_{---} = I + \frac{4}{9s}; \quad a_{qqq} = \frac{4}{3s}; \quad a_{--q} = -\frac{2}{s}; \quad a_{-qq} = \frac{2}{3s}. \tag{A3}$$

Using these expressions one obtains:

$$
\begin{aligned}
\Delta_s \bar{v}(q_+) O_{\mu\nu} u(q_-) &= \frac{1}{16\pi} \sum_{spins} \int \frac{d\Omega}{2\pi} \frac{1}{2(q_-k)} \left[\bar{v}(q_+) \slashed{e}_2 (\slashed{q}_- - \slashed{k}) \slashed{e}_1 u(q_-) \right] \times \\
&\times \left[(k_1 k_2) e_1^\nu e_2^\mu + (e_1 e_2) k_1^\nu k_2^\mu - k_1^\nu e_2^\mu (e_1 k_2) - k_2^\mu e_1^\nu (e_2 k_1) \right] = \\
&= \frac{1}{16\pi} \left[\bar{v}(q_+) \gamma_\mu u(q_-) \right] q_{-\nu} \left[s(I - 2\alpha) - 4a_{g_-} + sa_{--} + sa_{q_-} + 2a_g \right] = \\
&= \frac{5}{72\pi} \left[\bar{v}(q_+) \gamma_\mu u(q_-) \right] q_{-\nu}.
\end{aligned}
\tag{A4}
$$

Performing the summation on the polarization states of $^3P_2$ quarkonium we obtain:

$$
\begin{aligned}
\sum_\chi \frac{1}{4} \mathrm{Sp} \left[\slashed{q}_+ \gamma_\mu \slashed{q}_- \gamma_\eta \right] &\otimes \frac{1}{4} \mathrm{Sp} \left[\left(\slashed{p}_+ + M_p \right) \gamma_\lambda \left(\slashed{p}_- - M_p \right) \Gamma_\eta \right] \chi_{\mu\nu} \chi_{\lambda\eta} = \\
&= \beta c s^2 M_p^2 \left(-\frac{1}{4} \right) \left(F_1 + \frac{s}{4M_p^2} F_2 \right) - \frac{1}{16} (s\beta c)^3 F_1.
\end{aligned}
\tag{A5}
$$

Appendix B: The angular integrals for proton tensor

The angular integrals used for description of conversion of $^3P_2$ state to proton and antiproton are reported here:

$$
\int \frac{d\Omega_1}{2\pi(1 - \beta c_1)} \left\{ 1; (1 - c_2); (1 - c_2)^2; (1 - c_2)^3 \right\} = \left\{ J_0; J_1; J_2; J_3 \right\},
\tag{B1}
$$

where

$$
d\Omega_1 = dc_1 \phi_1; \quad c_1 = \cos(\mathbf{k_1 p_+}); \quad c_2 = \cos(\mathbf{k_1 q_-}) = cc_1 - ss_1 \cos\phi_1; \quad c = \cos(\mathbf{q_- p_+}).
$$

The result of integrations is

$$
J_0 = \frac{1}{\beta} L, \ L = \ln \frac{1+\beta}{1-\beta};
$$

$$
J_1 = \int_{-1}^1 \frac{dc_1 (1 - cc_1)}{1 - \beta c_1};
$$

$$
J_2 = \int_{-1}^1 \frac{dc_1 [(1 - cc_1)^2 + (1 - c^2)(1 - c_1^2)/2]}{1 - \beta c_1};
$$

$$
J_3 = \int_{-1}^1 \frac{dc_1 [(1 - cc_1)^2 + 3(1 - c^2)(1 - c_1^2)/2]}{1 - \beta c_1}.
\tag{B2}
$$

Let us note that the integrand in (B1) contains factor $(1 - \beta c_1)$, where β is the proton velocity. This factor thus determines the lower limit of integration on E_1^2 in the dispersion integral used for real part of amplitude restoration (see (29)):

$$
H_{odd}(\beta, c) = \frac{\mathcal{P}}{2\pi} \int_{M_p^2}^\infty \frac{dE_1^2}{E_1^2 - E^2} G_{odd}(E_1, c).
\tag{B3}
$$

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